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Some subordination and superordination results of generalized Srivastava-Attiya operator

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Abstract

In this article, we obtain some subordination and superordination-preserving results of the generalized Srivastava-Attiya operator. Sandwich-type result is also obtained.

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1 Introduction

Let $H(U)$ be the class of functions analytic in $U = \{z \in \mathbb{C} : |z| < 1\}$ and $H[a, n]$ be the subclass of $H(U)$ consisting of functions of the form $f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \dots$, with $H_0 = H[0, 1]$ and $H = H[1, 1]$. Denote $A(p)$ by the class of all analytic functions of the form

$$f(z) = z^p + \sum_{n=1}^{\infty} a_{p+n} z^{p+n} \quad (p \in \mathbb{N} = \{1, 2, 3, \dots\}; z \in U) \quad (1.1)$$

and let $A(1) = A$. For $f, F \in H(U)$, the function $f(z)$ is said to be subordinate to $F(z)$, or $F(z)$ is superordinate to $f(z)$, if there exists a function $\omega(z)$ analytic in U with $\omega(0) = 0$ and $|\omega(z)| < 1$ ($z \in U$), such that $f(z) = F(\omega(z))$. In such a case we write $f(z) \prec F(z)$. If F is univalent, then $f(z) \prec F(z)$ if and only if $f(0) = F(0)$ and $f(U) \subset F(U)$ (see [1,2]).

Let $\phi : \mathbb{C}^2 \times U \rightarrow \mathbb{C}$ and $h(z)$ be univalent in U . If $p(z)$ is analytic in U and satisfies the first order differential subordination:

$$\phi(p(z), zp'(z); z) \prec h(z), \quad (1.2)$$

then $p(z)$ is a solution of the differential subordination (1.2). The univalent function $q(z)$ is called a dominant of the solutions of the differential subordination (1.2) if $p(z) \prec q(z)$ for all $p(z)$ satisfying (1.2). A univalent dominant \tilde{q} that satisfies $\tilde{q} \prec q$ for all dominants of (1.2) is called the best dominant. If $p(z)$ and $\phi(p(z), zp'(z); z)$ are univalent in U and if $p(z)$ satisfies the first order differential superordination:

$$h(z) \prec \phi(p(z), zp'(z); z), \quad (1.3)$$

then $p(z)$ is a solution of the differential superordination (1.3). An analytic function $q(z)$ is called a subinvariant of the solutions of the differential superordination (1.3) if q

$(z) \prec p(z)$ for all $p(z)$ satisfying (1.3). A univalent subordinator \tilde{q} that satisfies $q \prec \tilde{q}$ for all subordinants of (1.3) is called the best subordinator (see [1,2]).

The general Hurwitz-Lerch Zeta function $\Phi(z, s, a)$ is defined by:

$$\Phi(z, s, a) = \sum_{n=0}^{\infty} \frac{z^n}{(n+a)^s}, \quad (1.4)$$

$(a \in \mathbb{C} \setminus \mathbb{Z}_0^-; \mathbb{Z}_0^- = \{0, -1, -2, \dots\}; s \in \mathbb{C} \text{ when } |z| < 1; R\{s\} > 1 \text{ when } |z| = 1)$.

For further interesting properties and characteristics of the Hurwitz-Lerch Zeta function $\Phi(z, s, a)$ (see [3-7]).

Recently, Srivastava and Attiya [8] introduced the linear operator $L_{s,b} : A \rightarrow A$, defined in terms of the Hadamard product by

$$L_{s,b}(f)(z) = G_{s,b}(z) * f(z) \quad (z \in U; b \in \mathbb{C} \setminus \mathbb{Z}_0^-; s \in \mathbb{C}), \quad (1.5)$$

where

$$G_{s,b} = (1+b)^s [\Phi(z, s, b) - b^{-s}] \quad (z \in U). \quad (1.6)$$

The Srivastava-Attiya operator $L_{s,b}$ contains among its special cases, the integral operators introduced and investigated by Alexander [9], Libera [10] and Jung et al. [11].

Analogous to $L_{s,b}$, Liu [12] defined the operator $J_{p,s,b} : A(p) \rightarrow A(p)$ by

$$J_{p,s,b}(f)(z) = G_{p,s,b}(z) * f(z) \quad (z \in U; b \in \mathbb{C} \setminus \mathbb{Z}_0^-; s \in \mathbb{C}; p \in \mathbb{N}), \quad (1.7)$$

where

$$G_{p,s,b} = (1+b)^s [\Phi_p(z, s, b) - b^{-s}]$$

and

$$\Phi_p(z, s, b) = \frac{1}{b^s} + \sum_{n=0}^{\infty} \frac{z^{n+p}}{(n+1+b)^s}. \quad (1.8)$$

It is easy to observe from (1.7) and (1.8) that

$$J_{p,s,b}(f)(z) = z^p + \sum_{n=1}^{\infty} \left(\frac{1+b}{n+1+b} \right)^s a_{n+p} z^{n+p}. \quad (1.9)$$

We note that

(i) $J_{p,0,b}(f)(z) = f(z)$;

(ii) $J_{1,s,b}(f)(z) = L_{s,b}f(z)$ ($s \in \mathbb{C}, b \in \mathbb{C} \setminus \mathbb{Z}_0^-$), where the operator $L_{s,b}$ was introduced by Srivastava and Attiya [8];

(iii) $J_{p,1,v+p-1}(f)(z) = F_{v,p}(f(z))$ ($v > -p, p \in \mathbb{N}$), where the operator $F_{v,p}$ was introduced by Choi et al. [13];

(iv) $J_{p,\alpha,p}(f)(z) = I_p^\alpha f(z)$ ($\alpha \geq 0, p \in \mathbb{N}$), where the operator I_p^α was introduced by Shams et al. [14];

(v) $J_{p,m,p-1}(f)(z) = J_p^m f(z)$ ($m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}, p \in \mathbb{N}$), where the operator J_p^m was introduced by El-Ashwah and Aouf [15];

(vi) $J_{p,m,p+l-1}(f)(z) = J_p^m(l)f(z)$ ($m \in \mathbb{N}_0$, $p \in \mathbb{N}$, $l \geq 0$), where the operator $J_p^m(l)$ was introduced by El-Ashwah and Aouf [15].

It follows from (1.9) that:

$$z(J_{p,s+1,b}(f)(z))' = (b+1)J_{p,s,b}(f)(z) - (b+1-p)J_{p,s+1,b}(f)(z). \quad (1.10)$$

To prove our results, we need the following definitions and lemmas.

Definition 1 [1] Denote by \mathcal{F} the set of all functions $q(z)$ that are analytic and injective on $\bar{U} \setminus E(q)$ where

$$E(q) = \left\{ \zeta \in \partial U : \lim_{z \rightarrow \zeta} q(z) = \infty \right\}$$

and are such that $q'(\zeta) \neq 0$ for $\zeta \in \partial U \setminus E(q)$. Further let the subclass of \mathcal{F} for which $q(0) = a$ be denoted by $\mathcal{F}(a)$, $\mathcal{F}(0) \equiv \mathcal{F}_0$ and $\mathcal{F}(1) \equiv \mathcal{F}_1$.

Definition 2 [2] A function $L(z, t)$ ($z \in U$, $t \geq 0$) is said to be a subordination chain if $L(z, t)$ is analytic and univalent in U for all $t \geq 0$, $L(z, 0)$ is continuously differentiable on $[0; 1)$ for all $z \in U$ and $L(z, t_1) \prec L(z, t_2)$ for all $0 \leq t_1 \leq t_2$.

Lemma 1 [16] The function $L(z, t) : U \times [0; 1) \rightarrow \mathbb{C}$ of the form

$$L(z, t) = a_1(t)z + a_2(t)z^2 + \cdots \quad (a_1(t) \neq 0; t \geq 0)$$

and $\lim_{t \rightarrow \infty} |a_1(t)| = \infty$ is a subordination chain if and only if

$$\operatorname{Re} \left\{ \frac{z \partial L(z, t) / \partial z}{\partial L(z, t) / \partial t} \right\} > 0 \quad (z \in U, t \geq 0).$$

Lemma 2 [17] Suppose that the function $\mathcal{H} : \mathbb{C}^2 \rightarrow \mathbb{C}$ satisfies the condition

$$\operatorname{Re} \{ \mathcal{H}(is; t) \} \leq 0$$

for all real s and for all $t \leq -n(1+s^2)/2$, $n \in \mathbb{N}$. If the function $p(z) = 1 + p_n z^n + p_{n+1} z^{n+1} + \dots$ is analytic in U and

$$\operatorname{Re} \{ \mathcal{H}(p(z); zp'(z)) \} > 0 \quad (z \in U),$$

then $\operatorname{Re} \{ p(z) \} > 0$ for $z \in U$.

Lemma 3 [18] Let $\kappa, \gamma \in \mathbb{C}$ with $\kappa \neq 0$ and let $h \in H(U)$ with $h(0) = c$. If $\operatorname{Re} \{ \kappa h(z) + \gamma \} > 0$ ($z \in U$), then the solution of the following differential equation:

$$q(z) + \frac{zq'(z)}{\kappa q(z) + \gamma} = h(z) \quad (z \in U; q(0) = c)$$

is analytic in U and satisfies $\operatorname{Re} \{ \kappa q(z) + \gamma \} > 0$ for $z \in U$.

Lemma 4 [1] Let $p \in \mathcal{F}(a)$ and let $q(z) = a + a_n z^n + a_{n+1} z^{n+1} + \dots$ be analytic in U with $q(z) \neq a$ and $n \geq 1$. If q is not subordinate to p , then there exists two points $z_0 = r_0 e^{i\theta} \in U$ and $\zeta_0 \in \partial U \setminus E(q)$ such that

$$q(U_{r_0}) \subset p(U); \quad q(z_0) = p(\zeta_0) \text{ and } z_0 p'(z_0) = m \zeta_0 p'(\zeta_0) (m \geq n).$$

Lemma 5 [2] Let $q \in H[a; 1]$ and $\phi : \mathbb{C}^2 \rightarrow \mathbb{C}$. Also set $\phi(q(z), zq'(z)) = h(z)$. If $L(z, t) = \phi(q(z), tzq'(z))$ is a subordination chain and $q \in H[a; 1] \cap \mathcal{F}(a)$, then

$$h(z) \prec \varphi(q(z), zq'(z)),$$

implies that $q(z) \prec p(z)$. Furthermore, if $\phi(q(z), zq'(z)) = h(z)$ has a univalent solution $q \in \mathcal{F}(a)$, then q is the best subordinator.

In the present article, we aim to prove some subordination-preserving and superordination-preserving properties associated with the integral operator $J_{p,s,b}$. Sandwich-type result involving this operator is also derived.

2 Main results

Unless otherwise mentioned, we assume throughout this section that $b \in \mathbb{C} \setminus \mathbb{Z}_0^-$, $s \in \mathbb{C}$, $\operatorname{Re}\{b\}$, $\mu > 0$, $p \in \mathbb{N}$, $z \in \mathbb{U}$ and the powers are understood as principle values.

Theorem 1. Let $f, g \in A(p)$ and

$$\operatorname{Re} \left\{ 1 + \frac{z\phi''(z)}{\phi'(z)} \right\} > -\delta \left(\phi(z) = \left(\frac{J_{p,s-1,b}(g)(z)}{J_{p,s,b}(g)(z)} \right) \left(\frac{J_{p,s,b}(g)(z)}{z^p} \right)^\mu; z \in U \right), \quad (2.1)$$

where δ is given by

$$\delta = \frac{1 + \mu^2|b+1|^2 - |1 - \mu^2(b+1)|^2}{4\mu[1 + \operatorname{Re}\{b\}]} (z \in U). \quad (2.2)$$

Then the subordination condition

$$\left(\frac{J_{p,s-1,b}(f)(z)}{J_{p,s,b}(f)(z)} \right) \left(\frac{J_{p,s,b}(f)(z)}{z^p} \right)^\mu \prec \left(\frac{J_{p,s-1,b}(g)(z)}{J_{p,s,b}(g)(z)} \right) \left(\frac{J_{p,s,b}(g)(z)}{z^p} \right)^\mu, \quad (2.3)$$

implies that

$$\left(\frac{J_{p,s,b}(f)(z)}{z^p} \right)^\mu \prec \left(\frac{J_{p,s,b}(g)(z)}{z^p} \right)^\mu, \quad (2.4)$$

where $\left(\frac{J_{p,s,b}(g)(z)}{z^p} \right)^\mu$ is the best dominant.

Proof. Let us define the functions $F(z)$ and $G(z)$ in U by

$$F(z) = \left(\frac{J_{p,s,b}(f)(z)}{z^p} \right)^\mu \text{ and } G(z) = \left(\frac{J_{p,s,b}(g)(z)}{z^p} \right)^\mu \quad (z \in U) \quad (2.5)$$

and without loss of generality we assume that $G(z)$ is analytic, univalent on \bar{U} and

$$G'(\zeta) \neq 0 \quad (|\zeta| = 1).$$

If not, then we replace $F(z)$ and $G(z)$ by $F(\rho z)$ and $G(\rho z)$, respectively, with $0 < \rho < 1$. These new functions have the desired properties on \bar{U} , so we can use them in the proof of our result and the results would follow by letting $\rho \rightarrow 1$.

We first show that, if

$$q(z) = 1 + \frac{zG''(z)}{G'(z)} (z \in U), \quad (2.6)$$

then

$$\operatorname{Re}\{q(z)\} > 0 \quad (z \in U).$$

From (1.10) and the definition of the functions G, ϕ , we obtain that

$$\phi(z) = G(z) + \frac{zG'(z)}{\mu(b+1)}. \quad (2.7)$$

Differentiating both sides of (2.7) with respect to z yields

$$\phi'(z) = \left(1 + \frac{1}{\mu(b+1)}\right) G'(z) + \frac{zG''(z)}{\mu(b+1)}. \quad (2.8)$$

Combining (2.6) and (2.8), we easily get

$$1 + \frac{z\phi''(z)}{\phi'(z)} = q(z) + \frac{zq'(z)}{q(z) + \mu(b+1)} = h(z) \quad (z \in U). \quad (2.9)$$

It follows from (2.1) and (2.9) that

$$\operatorname{Re}\{h(z) + \mu(b+1)\} > 0 \quad (z \in U). \quad (2.10)$$

Moreover, by using Lemma 3, we conclude that the differential Equation (2.9) has a solution $q(z) \in H(U)$ with $h(0) = q(0) = 1$. Let

$$\mathcal{H}(u, v) = u + \frac{v}{u + \mu(b+1)} + \delta,$$

Where δ is given by (2.2). From (2.9) and (2.10), we obtain $\operatorname{Re}\{\mathcal{H}(q(z); zq'(z))\} > 0 \quad (z \in U)$.

To verify the condition

$$\operatorname{Re}\{\mathcal{H}(i\vartheta; t)\} \leq 0 \quad \left(\vartheta \in \mathbb{R}; t \leq -\frac{1+\vartheta^2}{2}\right), \quad (2.11)$$

we proceed as follows:

$$\begin{aligned} \operatorname{Re}\{\mathcal{H}(i\vartheta; t)\} &= \operatorname{Re}\left\{i\vartheta + \frac{t}{\mu(b+1) + i\vartheta} + \delta\right\} = \frac{t\mu(1 + \operatorname{Re}(b))}{|\mu(b+1) + i\vartheta|^2} + \delta \\ &\leq -\frac{\Upsilon(b, \vartheta, \delta)}{2|\mu(b+1) + i\vartheta|^2}, \end{aligned}$$

where

$$\Upsilon(b, \vartheta, \delta) = [\mu(1 + \operatorname{Re}(b)) - 2\delta]\vartheta^2 - 4\delta\mu\operatorname{Im}(b)\vartheta - 2\delta|\mu(b+1)|^2 + \mu(1 + \operatorname{Re}(b)). \quad (2.12)$$

For δ given by (2.2), the coefficient of ϑ^2 in the quadratic expression $\Upsilon(b, \vartheta, \delta)$ given by (2.12) is positive or equal to zero. To check this, put $\mu(b+1) = c$, so that

$$\mu(1 + \operatorname{Re}(b)) = c_1 \text{ and } \mu\operatorname{Im}(b) = c_2.$$

We thus have to verify that

$$c_1 - 2\delta \geq 0,$$

or

$$c_1 \geq 2\delta = \frac{1 + |c|^2 - |1 - c^2|}{2c_1}.$$

This inequality will hold true if

$$2c_1^2 + |1 - c^2| \geq 1 + |c|^2 = 1 + c_1^2 + c_2^2,$$

that is, if

$$|1 - c^2| \geq 1 - \operatorname{Re}(c^2),$$

which is obviously true. Moreover, the quadratic expression $\Upsilon(b, \vartheta, \delta)$ by ϑ in (2.12) is a perfect square for the assumed value of δ given by (2.2). Hence we see that (2.11) holds. Thus, by Lemma 2, we conclude that

$$\operatorname{Re}\{q(z)\} > 0 \quad (z \in U),$$

that is, that G defined by (2.5) is convex (univalent) in U . Next, we prove that the subordination condition (2.3) implies that

$$F(z) \prec G(z),$$

for the functions F and G defined by (2.5). Consider the function $L(z, t)$ given by

$$L(z, t) = G(z) + \frac{(1+t)zG'(z)}{\mu(b+1)} \quad (0 \leq t < \infty; z \in U). \quad (2.13)$$

We note that

$$\left. \frac{\partial L(z, t)}{\partial z} \right|_{z=0} = G'(0) \left(1 + \frac{1+t}{\mu(b+1)} \right) \neq 0 \quad (0 \leq t < \infty; z \in U; \operatorname{Re}\{\mu(b+1)\} > 0).$$

This show that the function

$$L(z, t) = a_1(t)z + \dots$$

satisfies the condition $a_1(t) \neq 0$ ($0 \leq t < \infty$). Further, we have

$$\operatorname{Re} \left\{ \frac{z \partial L(z, t) / \partial z}{\partial L(z, t) / \partial t} \right\} = \operatorname{Re}\{\mu(b+1) + (1+t)q(z)\} > 0 \quad (0 \leq t < \infty; z \in U).$$

Since $G(z)$ is convex and $\operatorname{Re}\{\mu(b+1)\} > 0$. Therefore, by using Lemma 1, we deduce that $L(z, t)$ is a subordination chain. It follows from the definition of subordination chain that

$$\phi(z) = G(z) + \frac{zG'(z)}{\mu(b+1)} = L(z, 0)$$

and

$$L(z, 0) \prec L(z, t) \quad (0 \leq t < \infty),$$

which implies that

$$L(\zeta, t) \notin L(U, 0) = \phi(U) \quad (0 \leq t < \infty; \zeta \in \partial U). \quad (2.14)$$

If F is not subordinate to G , by using Lemma 4, we know that there exist two points $z_0 \in U$ and $\zeta_0 \in \partial U$ such that

$$F(z_0) = G(\zeta_0) \text{ and } z_0 F'(z_0) = (1+t)\zeta_0 G'(\zeta_0) \quad (0 \leq t < \infty). \quad (2.15)$$

Hence, by using (2.5), (2.13), (2.15) and (2.3), we have

$$\begin{aligned} L(\zeta_0, t) &= G(\zeta_0) + \frac{(1+t)\zeta_0 G'(\zeta_0)}{\mu(b+1)} = F(z_0) + \frac{z_0 F'(z_0)}{\mu(b+1)} \\ &= \left(\frac{J_{p,s-1,b}(f)(z)}{J_{p,s,b}(f)(z)} \right) \left(\frac{J_{p,s,b}(f)(z)}{z^p} \right)^\mu \in \phi(U). \end{aligned}$$

This contradicts (2.14). Thus, we deduce that $F < G$. Considering $F = G$, we see that the function G is the best dominant. This completes the proof of Theorem 1.

We now derive the following superordination result.

Theorem 2. Let $f, g \in A(p)$ and

$$\operatorname{Re} \left\{ 1 + \frac{z\phi''(z)}{\phi'(z)} \right\} > -\delta \left(\phi(z) = \left(\frac{J_{p,s-1,b}(g)(z)}{J_{p,s,b}(g)(z)} \right) \left(\frac{J_{p,s,b}(g)(z)}{z^p} \right)^\mu; z \in U \right), \quad (2.16)$$

where δ is given by (2.2). If the function $\left(\frac{J_{p,s-1,b}(f)(z)}{J_{p,s,b}(f)(z)} \right) \left(\frac{J_{p,s,b}(f)(z)}{z^p} \right)^\mu$ is univalent in U and $\left(\frac{J_{p,s,b}(f)(z)}{z^p} \right)^\mu \in \mathcal{F}$, then the superordination condition

$$\left(\frac{J_{p,s-1,b}(g)(z)}{J_{p,s,b}(g)(z)} \right) \left(\frac{J_{p,s,b}(g)(z)}{z^p} \right)^\mu < \left(\frac{J_{p,s-1,b}(f)(z)}{J_{p,s,b}(f)(z)} \right) \left(\frac{J_{p,s,b}(f)(z)}{z^p} \right)^\mu, \quad (2.17)$$

implies that

$$\left(\frac{J_{p,s,b}(g)(z)}{z^p} \right)^\mu < \left(\frac{J_{p,s,b}(f)(z)}{z^p} \right)^\mu, \quad (2.18)$$

where $\left(\frac{J_{p,s,b}(f)(z)}{z^p} \right)^\mu$ is the best subdominant.

Proof. Suppose that the functions F, G and q are defined by (2.5) and (2.6), respectively. By applying similar method as in the proof of Theorem 1, we get

$$\operatorname{Re}\{q(z)\} > 0 \quad (z \in U).$$

Next, to arrive at our desired result, we show that $G < F$. For this, we suppose that the function $L(z, t)$ be defined by (2.13). Since G is convex, by applying a similar method as in Theorem 1, we deduce that $L(z, t)$ is subordination chain. Therefore, by using Lemma 5, we conclude that $G < F$. Moreover, since the differential equation

$$\phi(z) = G(z) + \frac{zG'(z)}{\mu(b+1)} = \varphi(G(z), zG'(z))$$

has a univalent solution G , it is the best subdominant. This completes the proof of Theorem 2.

Combining the above-mentioned subordination and superordination results involving the operator $J_{p,s,b}$, the following “sandwich-type result” is derived.

Theorem 3. Let $f, g_j \in A(p)$ ($j = 1, 2$) and

$$\operatorname{Re} \left\{ 1 + \frac{z\phi_j''(z)}{\phi_j'(z)} \right\} > -\delta \left(\phi_j(z) = \left(\frac{J_{p,s-1,b}(g_j)(z)}{J_{p,s,b}(g_j)(z)} \right) \left(\frac{J_{p,s,b}(g_j)(z)}{z^p} \right)^\mu \quad (j = 1, 2); z \in U \right),$$

where δ is given by (2.2). If the function $\left(\frac{J_{p,s-1,b}(f)(z)}{J_{p,s,b}(f)(z)}\right)\left(\frac{J_{p,s,b}(f)(z)}{z^p}\right)^\mu$ is univalent in U and $\left(\frac{J_{p,s,b}(f)(z)}{z^p}\right)^\mu \in \mathcal{F}$, then the condition

$$\begin{aligned} \left(\frac{J_{p,s-1,b}(g_1)(z)}{J_{p,s,b}(g_1)(z)}\right)\left(\frac{J_{p,s,b}(g_1)(z)}{z^p}\right)^\mu &< \left(\frac{J_{p,s-1,b}(f)(z)}{J_{p,s,b}(f)(z)}\right)\left(\frac{J_{p,s,b}(f)(z)}{z^p}\right)^\mu \\ &< \left(\frac{J_{p,s-1,b}(g_2)(z)}{J_{p,s,b}(g_2)(z)}\right)\left(\frac{J_{p,s,b}(g_2)(z)}{z^p}\right)^\mu, \end{aligned} \quad (2.19)$$

implies that

$$\left(\frac{J_{p,s,b}(g_1)(z)}{z^p}\right)^\mu < \left(\frac{J_{p,s,b}(f)(z)}{z^p}\right)^\mu < \left(\frac{J_{p,s,b}(g_2)(z)}{z^p}\right)^\mu, \quad (2.20)$$

where $\left(\frac{J_{p,s,b}(g_1)(z)}{z^p}\right)^\mu$ and $\left(\frac{J_{p,s,b}(g_2)(z)}{z^p}\right)^\mu$ are, respectively, the best subdominant and the best dominant.

Remark. (i) Putting $\mu = 1$, $b = p$ and $s = \alpha$ ($\alpha = 0$, $p \in \mathbb{N}$) in our results of this article, we obtain the results obtained by Aouf and Seoudy [19];

(ii) Specializing the parameters s and b in our results of this article, we obtain the results for the corresponding operators $F_{v,p}$, I_p^α , J_p^m and $J_p^m(l)$ which are defined in the introduction.

Authors' contributions

All authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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